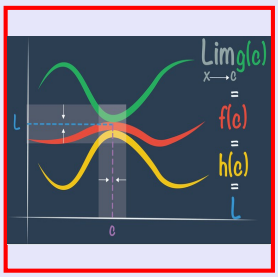
Math 261

Spring 2023

Lecture 47



Feb 19-8:47 AM

$$\int_0^1 x^2\sqrt{x+1}\,dx$$

$u = \sqrt{x+1}$

$u^2 = x+1 \rightarrow u^2 - 1 = x$

$2u\,du = dx$

$x = 0 \rightarrow u = 1$

$x = 1 \rightarrow u = \sqrt{2}$

$$\int_1^{\sqrt{2}} (u^2-1)^2 \cdot u \cdot 2u\,du$$

$$= 2\int_1^{\sqrt{2}} (u^4 - 2u^2 + 1)u^2\,du$$

$$= 2\int_1^{\sqrt{2}} (u^6 - 2u^4 + u^2)\,du$$

$$= 2\left[\frac{u^7}{7} - \frac{2u^5}{5} + \frac{u^3}{3}\right]_1^{\sqrt{2}}$$

$$= 2\left[\left(\frac{(\sqrt{2})^7}{7} - \frac{2(\sqrt{2})^5}{5} + \frac{(\sqrt{2})^3}{3}\right) - \left(\frac{1}{7} - \frac{2}{5} + \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{(\sqrt{2})^6 \cdot \sqrt{2}}{7} - \frac{2(\sqrt{2})^4\sqrt{2}}{5} + \frac{(\sqrt{2})^2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$$

$$= 2\left[\frac{8\sqrt{2}}{7} - \frac{8\sqrt{2}}{5} + \frac{2\sqrt{2}}{3} - \frac{1}{7} + \frac{2}{5} - \frac{1}{3}\right]$$

$$= 2\left[\frac{15 \cdot 8\sqrt{2} - 21 \cdot 8\sqrt{2} + 35 \cdot 2\sqrt{2} - 15 + 21 \cdot 2 - 35}{105}\right]$$

$$= 2\left(\frac{120\sqrt{2} - 168\sqrt{2} + 70\sqrt{2} - 15 + 42 - 35}{105}\right)$$

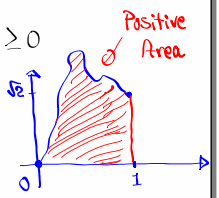
$$= 2\left(\frac{22\sqrt{2} - 8}{105}\right) = \boxed{\frac{44\sqrt{2} - 16}{105}} > 0$$

May 9-9:45 AM

$$\int_0^1 x^2\sqrt{x+1}\,dx$$

$$f(x) = \underset{+}{x^2}\underset{+}{\sqrt{x+1}} \geq 0$$

$$f(0)=0 \quad , \quad f(1)=\sqrt{2}$$



Evaluate $\int_0^1 x^3\sqrt{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx \rightarrow \frac{du}{2} = dx$

$x=0 \rightarrow u=1$

$x=1 \rightarrow u=2$

$$\int_0^1 x^3\sqrt{x^2+1}\,dx =$$

$$\int_0^1 x^2 \cdot x\sqrt{x^2+1}\,dx = \int_1^2 (u-1)\sqrt{u}\,\frac{du}{2}$$

$$= \int_1^2 \frac{1}{2}\left[u\sqrt{u} - \sqrt{u}\right]du = \frac{1}{2}\int_1^2 \left(u^{3/2} - u^{1/2}\right)du$$

$$= \frac{1}{2}\left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]\Bigg]_1^2 = \left(\frac{1}{5}u^2\sqrt{u} - \frac{1}{3}u\sqrt{u}\right)\Bigg]_1^2$$

$$= \left(\frac{1}{5}\cdot 4\sqrt{2} - \frac{1}{3}\cdot 2\sqrt{2}\right) - \left(\frac{1}{5} - \frac{1}{3}\right) = \frac{4\sqrt{2}}{5} - \frac{2\sqrt{2}}{3} - \frac{1}{5} + \frac{1}{3}$$

$$= \frac{12\sqrt{2} - 10\sqrt{2} - 3 + 5}{15} = \boxed{\frac{2\sqrt{2}+2}{15}} \checkmark$$

May 10-8:48 AM

$$\int_0^1 x^3\sqrt{x^2+1}\,dx =$$

$u = \sqrt{x^2+1}$

$u^2 = x^2+1$

$\cancel{2}u\,du = \cancel{2}x\,dx \rightarrow u\,du = x\,dx$

$$\int_0^1 x^3\sqrt{x^2+1}\,dx =$$

$x=0 \qquad u=1$

$x=1 \qquad u=\sqrt{2}$

$$\int_0^1 x^2 \cdot x \cdot \sqrt{x^2+1}\,dx = \int_1^{\sqrt{2}} (u^2-1)\cdot u \cdot u\,du$$

$$= \int_1^{\sqrt{2}} (u^4 - u^3)\,du = \left(\frac{u^5}{5} - \frac{u^3}{3}\right)\Bigg]_1^{\sqrt{2}}$$

$$= \left(\frac{(\sqrt{2})^5}{5} - \frac{(\sqrt{2})^3}{3}\right) - \left(\frac{1}{5} - \frac{1}{3}\right)$$

$$= \frac{(\sqrt{2})^4\sqrt{2}}{5} - \frac{(\sqrt{2})^2\sqrt{2}}{3} - \frac{1}{5} + \frac{1}{3}$$

$$= \frac{4\sqrt{2}}{5} - \frac{2\sqrt{2}}{3} - \frac{1}{5} + \frac{1}{3}$$

$$= \frac{12\sqrt{2} - 10\sqrt{2} - 3 + 5}{15} = \boxed{\frac{2\sqrt{2}+2}{15}}$$

May 10-9:07 AM

Find the area below $f(x)=(x-2)^2$, above the x-axis from $x=0$ to $x=4$.

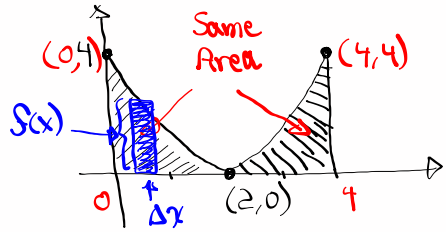
$f(x)=(x-2)^2$

Parabola

Vertex $(2,0)$

$f(0)=4$

$f(4)=4$



$$A=2\int_0^2 (x-2)^2\,dx$$

$u=x-2$

$du=dx$

$x=0 \to u=-2$

$x=2 \to u=0$

$$A=2\int_{-2}^{0} u^2\,du = 2\cdot\frac{u^3}{3}\Big]_{-2}^{0} = \frac{2}{3}\left[0^3-(-2)^3\right]$$

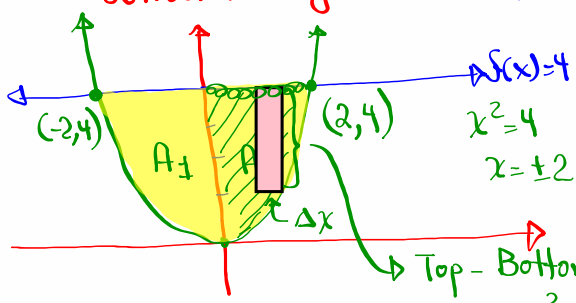
$$=\frac{2}{3}(8)=\boxed{\frac{16}{3}}$$

May 10-9:15 AM

Find the area enclosed by $\boxed{f(x)=4}$ and $\boxed{g(x)=x^2}$.

Surrounded by Horizontal line

Parabola open up $V(0,0)$



$x^2=4$

$x=\pm 2$

Top − Bottom

$4-x^2$

$$A=2\int_0^2 (4-x^2)\,dx$$

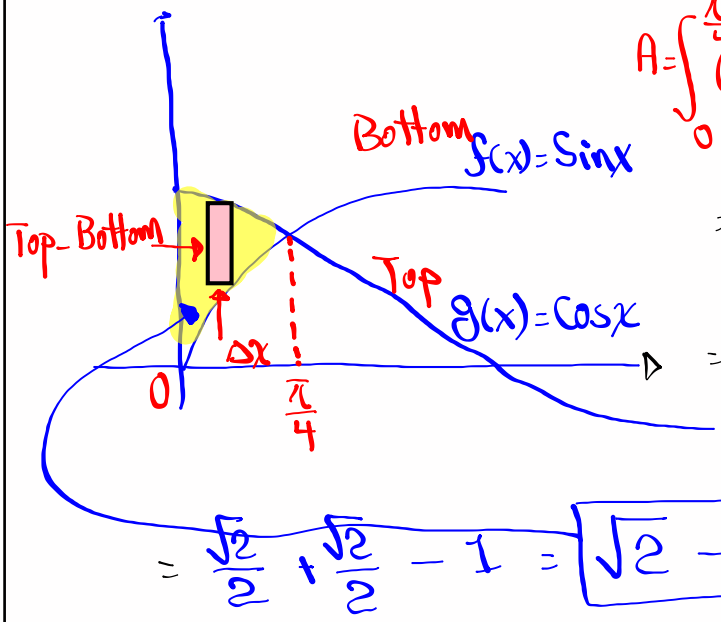
$$=2\left[4x-\frac{x^3}{3}\right]\Big]_0^2$$

$$=2\left[\left(4\cdot 2-\frac{2^3}{3}\right)-\left(4\cdot 0-\frac{0^3}{3}\right)\right] \quad 0$$

$$=2\left[8-\frac{8}{3}\right]=2\cdot\frac{16}{3}=\boxed{\frac{32}{3}}$$

May 10-9:23 AM

Find the area shaded below



$$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x)\,dx$$

$$= (\sin x + \cos x)\Big]_0^{\pi/4}$$

$$= \left(\sin\frac{\pi}{4} + \cos\frac{\pi}{4}\right) - \left(\cancel{\sin 0}^{\,0} + \cancel{\cos 0}^{\,1}\right)$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1 = \boxed{\sqrt{2} - 1}$$

May 10-9:31 AM

Suppose $f(x)$ is cont. and $\int_0^9 f(x)\,dx = 5$

Find $\int_0^3 x f(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{2} = x\,dx$

$x = 0 \rightarrow u = 0$

$x = 3 \rightarrow u = 9$

$$= \int_0^9 f(u)\frac{du}{2} =$$

$$= \frac{1}{2}\int_0^9 f(u)\,du$$

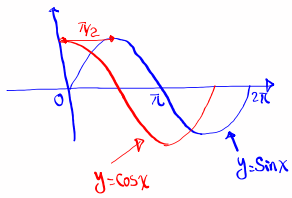
$$= \frac{1}{2} \cdot 5 = \boxed{\frac{5}{2}}$$

May 10-9:37 AM

Suppose $f(x)$ is cont., Prove

$$\int_0^{\pi/2} f(\cos x)\,dx = \int_0^{\pi/2} f(\sin x)\,dx$$

Bad Subs. $\begin{cases} u = \sin x & du = \cos x\,dx \\ x = 0 & u = 0 \\ x = \pi/2 & u = 1 \end{cases} \Rightarrow \int_0^{\pi/2} f(\sin x)\,dx = \int_0^1 f(u)\,?$



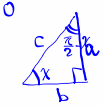
$u = \frac{\pi}{2} - x \qquad du = -dx \rightarrow -du = dx$

$x = 0 \rightarrow u = \frac{\pi}{2}$

$x = \frac{\pi}{2} \rightarrow u = 0$

$\sin u = \sin\left(\frac{\pi}{2} - x\right)$

$= \cos x$



$\cos x = \frac{b}{c} \qquad \sin\left(\frac{\pi}{2} - x\right) = \frac{b}{c}$

$$\int_0^{\pi/2} f(\cos x) = \int f(\sin u) \cdot -du$$

$$= -\int_{\pi/2}^{0} f(\sin u)\,du$$

Look this up

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

May 10-9:42 AM